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BASIC ELECTRICAL ENGINEERING BT-104 BEEE NOTES OF UNIT IV D.C. MACHINE DR. SYED UVAID ULLAH

Q.1 Classify the rotating electrical machine with their applications.

(R.G.P.V. Dec. 2010)

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- Ans. Basically electrical machine is two types
- (1) Static Machine

(I) Transformer

- (2) Rotating machine
 - (II) D.C. Machine
 - (i) D.C. Generator
 - (ii) D.C. Motor
 - (III) A.C. Machine
 - (i) Synchronous Machine(A) Synchronous Generator/Motor
 - (a) Single Phase
 - (b) Three phase
 - (ii) Asynchronous Machine (A) Induction Generator/Motor
 - (a) Single Phase
 - (b) Three phase

(i) Static Machine-

(a) Transformer- Transformer is a static device because it has no rotating part. Transformer is an electromagnetic device which works on faraday's law of electromagnetic induction.

Application- Industrial Machineries, Manufacturing Industry, Transformer used to provide required voltage level, Electrical Appliances etc.

(ii) D.C. Machines are of three types

(a) D.C. Series Machine- When field winding and armature winding are connected in series so is called D.C. series machine-

Application- Industrial uses are hoists, cranes, trolly cars, conveyors, elevators, air compressors, vacuum cleaners, sewing machines etc.

(b) D.C. shunt machine- When field winding and armature winding are connected in parallel so is called D.C. series machine.

Application-Centrifugal Pumps, Blowers, Conveyors, Lifts

(c) D.C. compound machine- D.C. compound machine is combination of both series and shunt D.C. machine. One field winding is connected in series with armature and second field winding is connected in parallel with armature.





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Application- A compound motor is used in Conveyors, Elevators, Rolling Mills, Heavy Planners, Presses, Shears, etc

(iii) A.C. Machines are of two types-

(a) Synchronous Machine- Synchronous motors are a doubly excited machine, two electrical inputs are provided to it. The rotor carrying dc supply also produces a constant flux. The main part of synchronous machine is stator and rotor there are two types of rotor used(i) Salient Pole Type

(ii) Cylindrical Type (Non- Salient Pole)

Application- Robot actuators. Ball mills, clocks, record player.

(ii) Induction motor - An induction motor also known as an asynchronous motor. Induction motors are referred to as 'asynchronous motors' because they operate at a speed less than their synchronous speed. Induction motor has two main part first is stator which is stationary and second is rotor is called rotating part.

Application- Induction motors are used in conveyors, cranes, pumps, elevators and compressors.

O.2 Write the necessity	and material used for the foll	owing in a D.C. machine-
(i) Commutator	(ii) Brush	(R.G.P.V. Dec. 2014)

Ans. **Commutator-** A Commutator is a electrical switch in specific types of electric motors and electrical generators that once in a while reverses the current direction between the rotor and the external circuit. The windings on the armature are connected to the Commutator segments.

Commentators are used in direct current (D.C.) machines reversing the direction of the current with each half turn, serving as a mechanical rectifier to convert the alternating current from the windings to unidirectional direct current in the external load circuit. It is of cylindrical structure it is built up of wedge shaped segment of high conductivity. Brushes mounted on Commutator surface for collection of current from the armature conductors

Brushes- Brushes are made of carbon or electrographite material this material is used to avoid wear and tear of Commutator. The function of brushes is to collect the current from Commutator. Brushes are generally rectangular in shape. They are housed in brush holder

Q.3 Describe the construction details of a D.C. Machine giving suitable diagram.

(R.G.P.V. Dec. 2013)

OR

Write down the constructional features of a D.C. machine with neat and suitable diagrams.

(R.G.P.V. June 2016)

OR

Describe D.C. machine with neat sketches in viewing of main parts and constructional details.

R.G.P.V. Dec. 2016, 2017)

OR

Name the main parts of a D.C. machine and indicate their functions.

(R.G.P.V. Dec. 2015)





Ans. Every rotating machine has two important part first is stationary part that is called stator and second is rotating part that is called rotor. In case of D.C. machine field winding is on the stator and armature winding is on the rotor.

The main part of D.C. machine

(i) Yoke- yoke is an outer frame of D.C. machine which consist of unlaminated ferromagnetic material. It provides mechanical support to the inner part of machine. It provides a path for magnetic flux. The yoke is prepared of cast iron for smaller machine and it is made of cast steel for larger machines.

(ii) Pole – Each pole divided into two parts

- (a) Pole core
- (b) Pole shoe

(iii) **Pole core**- Field winding is mounted on the pole core which is necessary to produce the flux. They extend out the magnetic flux over the armature outside edge more uniformly. Pole core is made of cast iron or cast steel. Pole core are fixed to the magnetic frame or yoke by bolts or by weld.

(iv) Pole shoes- Pole shoes are made of cast iron or cast steel. Pole shoes have larger cross sectional area as compare to pole core which provide less reluctance for magnetic path. The front segment of pole is of larger cross section area and known as the pole shoe or pole face. The pole shoe provide the following serves

- (a) Pole shoe makes the air gap flux uniform.
- (b) Pole shoe provide mechanical support to the field winding
- (c) Pole shoe decrease the reluctance of the magnetic path.

(v) Field winding- field winding is called exciting winding field winding is made from copper material. When D.C current is passed through the field winding it magnetizes the poles which create the required flux. The field coils of all the poles are associated in series in such a way that when current through them the nearby poles reach opposite polarity.

(vi) Armature – Armature further divided into two parts namely.

(a) Armature core- it is cylindrical in shape and mounded on the rotating shaft. It consists of slots on its circumference and the air ducts to allow the air flow from beginning to end armature which serves cooling purpose. It serves for housing armature coils in the slots and given that the low reluctance path to the magnetic flux. Since armature is a rotating part of the machine. To minimize hysteresis losses silicon steel material is used for its construction.

(b) Armature winding- the insulated conductors housed in the armature slots are properly connected. The armature winding is main part of D.C. machine. Armature winding is the interconnection of the armature conductor. In case of generator when armature is rotated magnetic flux get cut by armature conductor and an e.m.f. get induced in them. On the basis of connection armature winding is two types (i) Lap winding (ii) Wave winding.

(vii) Commutator- A Commutator is a electrical switch in specific types of electric motors and electrical generators that once in a while reverses the current direction between the rotor and the external circuit. The windings on the armature are connected to the Commutator segments.

Commentators are used in direct current (D.C.) machines reversing the direction of the current with each half turn, serving as a mechanical rectifier to convert the alternating current from the windings to unidirectional direct current in the external load circuit. It is of cylindrical structure it is built up of wedge shaped segment of high conductivity. Brushes mounted on Commutator surface for collection of current from the armature conductors





(viii) Brushes- Brushes are made of carbon or electrographite material this material is

used to avoid wear and tear of Commutator. The function of brushes is to collect the current from Commutator. brushes are generally rectangular in shape. They are housed in brush holder.

(ix) Bearing - The most important purpose of a bearing is to support the rotating parts and to permit its smooth motion with minimum friction. For medium size machines roller bearing may be used at the driving end and ball bearing at the non driving end. For small machines ball bearing are used at both ends. The main function of the bearing is to reduce friction between rotating part and stationary part.

(x) Shaft – The shaft is used to transfer mechanical power from one machine to other machine. The rotating part of D.C. machine like armature, Commutator, cooling fan are mounted on the shaft.

Q. 5 Classify self excited D.C. motor.

(R.G.P.V. Dec. 2014)

Ans. Self excited D.C. motor can be further classified as follow

(i) Series Motor – In case of D.C. series motor field winding is connected in series with armature winding.



(ii) **Shunt Motor** – In case of D.C. shunt motor field winding is connected in parallel with armature winding.

 $I_{L} = Load current$

 $I_a = Armature current$

 I_{sh} = Shunt Field winding current

V = Supply voltage

 $E_{\rm b} = \text{Back e.m.f.}$

 $R_a =$ Armature Resistance

 R_{se} = Series Field winding Resistance

 $V_{b} =$ Voltage drop across brushes

$$I_{L} = I_{a} + I_{sh}$$
$$V = E_{b} + I_{a}R_{a} + 2V_{b}$$

(iii) **Compound Motor** - D.C. compound machine is combination of both series and shunt D.C. machine. One field winding is connected in series with armature and second field







winding is connected in parallel with armature. The compound motor is two types. (a) Long shunt compound wound (b) Short shunt compound wound

 $I_{L} = Load$ current

 $I_a = Armature current$

 $I_{sh} = shunt Field winding current$

V = Supply voltage

 $E_{\rm b} = \text{Back e.m.f.}$

 $R_a =$ Armature Resistance

Short shunt compound wound

R_{sh}=Shunt Field winding Resistance

 R_{se} = Series Field winding Resistance

 $V_{\rm b} =$ Voltage drop across brushes



$$I_{L} = I_{a} + I_{sh}$$
$$V = E_{b} + I_{L}R_{se} + I_{a}R_{a} + 2V_{b}$$

(iv) **Compound Motor** – D.C. compound machine is combination of both series and shunt D.C. machine. One field winding is connected in series with armature and second field winding is connected in parallel with armature. The compound motor is two types. (a) Long shunt compound wound (b) Short shunt compound wound



Long shunt compound wound

 $R_{se} = Series$ Field winding Resistance

 $V_{b} =$ Voltage drop across brushes

$$I_{L} = I_{a} + I_{sh}$$
$$V = E_{b} + I_{se}R_{se} + I_{a}R_{a} + 2V_{b}$$

Q. 6 State the types of D.C. Motors. Discuss constructional details of and type of D.C. motor.

(R.G.P.V. June 2013)

Ans. Refer Q. No. 5 and Q. No. 3

Q. 7 What do you mean by separately excited and self excited D.C. generator sketch following type of D.C. generator-

(i) Shunt wound (ii) Series wound (iii) Compound generator. (R.G.P.V. Dec. 2013)





OR

Classify D.C. machines and explain them briefly.

(**R.G.P.V. June 2014**)

Ans. **Separately Excited D.C. Generator** – In case of separately excited D.C. generator a separate external D.C. supply is used to provide exciting current through the field winding. There is no electrical connection between field winding and armature winding.



 $E_{\rm h} = V + I_a R_a$

Circuit Globe

OAD

(i) **Series Generator** – In case of D.C. series generator field winding is connected in series with armature winding.

Series Field

- $I_{L} = Load current$
- $I_a = Armature current$

I_{se} = Series Field winding current

- $R_a = Armature Resistance$
- R_{se} = Series Field winding Resistance
- $V_{b} = Voltage drop across brushes$
- V = Terminal voltage

 $E_b = \text{Generated e.m.f. in volts OWARDS BEING THE BEST'}$ $I_L = I_a = I_{se}$



(ii) **Shunt Generator** – In case of D.C. shunt generator field winding is connected in parallel with armature winding.

- $I_L = Load current$
- $I_a = Armature current$
- I_{sh} = Shunt Field winding current
- I_{se} = Series Field winding current
- $R_a =$ Armature Resistance
- R_{sh}=Shunt Field winding Resistance
- R_{se} = Series Field winding Resistance
- $V_{b} =$ Voltage drop across brushes

V = Terminal voltage







 E_{b} = Generated e.m.f. in volts

$$I_{L} = I_{a} + I_{sh}$$
$$E_{b} = V + I_{a}R_{a} + I_{se}R_{se} + 2V_{b}$$

(iii) **Compound Generator** – D.C. compound machine is combination of both series and shunt D.C. machine. One field winding is connected in series with armature and second field winding is connected in parallel with armature. The compound generator is two types. (a) Long shunt compound wound (b) Short shunt compound wound

 $I_L = Load$ current

 $I_a = Armature current$

I_{sh} = Shunt Field winding current

 I_{se} =Series Field winding current

R_a = Armature Resistance

 R_{sh} = Shunt Field winding Resistance

 R_{se} = Series Field winding Resistance

wound

 $V_{b} =$ Voltage drop across brushes

V = Terminal voltage

 $E_{\rm b}$ = Generated e.m.f. in volts

$$I_{I} = I_{a} + I_{ch}$$

 $\mathbf{V} = \mathbf{E}_{\mathbf{b}} + \mathbf{I}_{\mathbf{se}}\mathbf{R}_{\mathbf{se}} + \mathbf{I}_{a}\mathbf{R}_{a} + 2\mathbf{V}_{\mathbf{b}}$





(iv) Compound Generator – D.C. compound machine is combination of both series and shunt D.C. machine. One field winding is connected in series with armature and second field winding is connected in parallel with armature. The compound generator is two types. (i) Long shunt compound wound (ii) Short shunt compound wound

 $I_{\rm L} = \text{Load current}$

- $I_a = Armature current$
- I_{sh} = Shunt Field winding current

V = Supply voltage

 $E_{b} = Back e.m.f.$

 $R_a = Armature Resistance$

R_{sh}=Shunt Field winding Resistance

R_{se}=Series Field winding Resistance

 $V_{b} =$ Voltage drop across brushes



Long shunt compound wound

$$I_{L} = I_{a} + I_{sh}$$
$$V = E_{b} + I_{se}R_{se} + I_{a}R_{a} + 2V_{b}$$



Q.8 Write the basic principle of operation and working of D.C. motor.

(R.G.P.V. June 2007, 2008, July 2008)

OR

Explain working principle of D.C. motor with necessary diagram. (R.G.P.V. June 2017)

OR

With a neat diagram explain the working and principle of D.C. motor.

(R.G.P.V. May 2019)

Ans. A machine which converts D.C. electrical power into mechanical power is called D.C. motor. This mechanical energy is utilized to drive mechanical load coupled to the shaft of the motor. Construction of D.C. motor is same as D.C. generator. The principle of operation of a D.C. motor can be declared in a single statement as when a current carrying conductor is placed in a magnetic field a mechanical force is occurrence by it. The direction of this force is definite by Fleming's left hand rule. If conductors is free to rotate then it starts rotation in the direction of force.

- $F = BIl \sin\theta$ Newton
- B = Flux density in wb/m²
- I = Current in Amp.
- 1 =length of conductor in meter
- θ = angle between field and conductor



For simplicity, consider that the armature of D.C. motor has only one coil which is located between the magnetic field shown in the figure A. When the D.C. supply is connected to the armature coil the current flowing through it. Which build up their own field around the coil as shown in Figure B



By the interaction of the two fields produces by the pole and the magnet, a resultant field set up across the conductor. The resultant field has a propensity to recover its original position, i.e. in the axis of the main field. The fields apply the force at the ends of the conductor, and consequently the coil starts rotating.

Q.9 Explain the working principle and construction of D.C. Machine.

(R.G.P.V. Dec. 2010)





Explain the constructional and operational feature of a D.C. machine with the help of neat diagram.

(R.G.P.V. Dec. 2012)

OR

Sate basic principle of D.C. motor. Draw diagram of D.C. machine and name its parts (R.G.P.V. May 2018)

Ans. Refer Q. No. 8 & 3

Q.10 Explain constructional classification and working principle of D.C. Machine.

(R.G.P.V. Nov. 2018)

Ans. Refer Q. No. 3, 5 & 8

Q.11 Drive the expression of generated voltage in D.C. machine.

(R.G.P.V. Dec. 2006, 2007-2011)

(R.G.P.V. Dec. 2016, 2017)

OR Drive e.m.f equation of a D.C. motor/generator. OR

Develop an e.m.f. equation for generator.

Ans. E.M.F. Equation of D.C. generator:

$$E = \frac{P\phi NP}{60A}$$

		OUA	
Р	=	Number of poles of generator	
ϕ	=	Flux produced by each pole in Weber	
Ν	. (=	Speed of armature in R.P.M.	h
Ζ		Total number of armature conductor	ł
A	=\/	Number of parallel path	-
Α	MAR	P For lap type of winding DDC REINC THE RECT	2
А	n çir	2 For wave type of winding	

Let E is the generated e.m.f. in the armature according to faraday's law of electromagnetic induction.

T = number of turns

$$E = T \frac{d\phi}{dt}$$

If number of turns T = 1 then

$$E = \frac{d\phi}{dt}$$

Now consider one revolution of conductor. In one revolution conductor will cut total flux produced by all the poles = $\phi \times P$

While time required to complete one revolution is 60/N seconds as speed in R.P.M.

$$E = \frac{P\phi}{\frac{60}{N}} = \frac{P\phi N}{60}$$

If total number of conductor = ZNumber of parallel path = A





Hence Z/A number of conductors are always in series and e.m.f. remains same across all the parallel paths.

The e.m.f. equation of a D.C. generator $E = \frac{P\phi N}{60} = \frac{P\phi N}{60} \times \frac{Z}{A} = \frac{P\phi NZ}{60A}$

Q. 12 Give reasons why starting current is high in D.C. motor? (R.G.P.V. June 2012)

Why you need a starter to start a D.C. motor?

(R.G.P.V. June 2015)

Ans. – A D.C. motor generate back e.m.f. which oppose the supply voltage as per Lenz's law so starting current reduce drawn by motor. At the time of starting motor is stationary mode and there is no back e.m.f in the armature. If motor is directly connect to supply voltage armature draw a heavy current because armature has small resistance and no back e.m.f. As an example 3 H.P, 200 V shunt motor has a full load current of 10A and armature resistance of about 0.5 ohm. If this motor is directly switched on to supply, so armature current is equal 200

to $\frac{200}{0.5} = 400A$ which is 40 times greater than the full load current. If speed of D.C. motor is

high so back e.m.f is high and armature current will low. If speed of D.C. motor is low so back e.m.f is low and armature current will high which develop motor torque.

Q. 13 What do you understand by commutation?

(R.G.P.V. Jan/Feb. 2007)

Ans- Commutation mean reversal of armature current from its positive value to same negative value by means of brushes and Commutator bars. In D.C. motors commutation is easy to identify with as brushes contact a Commutator and switch the current as the motor moves.

When current is functional to the motor, the accurate winding is energized by virtue of the brushes being in contact with the Commutator at the point where the winding terminates. As the motor moves, the next coil in the sequence will be energized. For collecting and feeding the electrical supply carbon or electrographite brushes are all time fitted on the Commutator surface. These brushes touch the Commutator surface and do not rotate with Commutator. With the help of commutation method induced e.m.f. in the armature conductors of a D.C. generator can be made unidirectional.

Q.14 what is a function of back e.m.f. in a D.C. motor? (R.G.P.V. Jan/Feb. 2007) OR

Give reason why induced e.m.f in a D.C. motor is called back e.m.f.

(R.G.P.V. Jan. /Feb. 2007)

Ans.- D.C. motor have two important part stator and armature. When motor armature rotates then armature conductors cut the magnetic flux. An e.m.f. induced in armature conductors by faraday's law of electromagnetic induction. Whose direction as found by Fleming's right hand rule the polarity of this induced e.m.f. is such that it can oppose the applied voltage in the armature circuit.

Because of it opposite direction it is called back e.m.f. and denoted by E_b . Mainly it obtain generated by the generating action which we have observe previous in case of generators. So it magnitude can be determined by the e.m.f. equation which is derived earlier







As per equivalent circuit diagram of D.C. motor the applied voltage always greater than the back e.m.f.

$$E_b = V - I_a R_a$$

V = supply voltage in volts.Ia = armature current in amp. Ra = armature resistance in ohm. E_b = back e.m.f. or counter e.m.f. in volts

Q. 16 What is Torque? What is the source of torque in D.C. motor?

(R.G.P.V. Jan. / Feb. 2007)

SM

Pulley

N r.p.s.

Ans. It is observe that turning or twisting force with reference to an axis is called torque. It is measured by the product of the force and radius. Consider a wheel of r meter acted upon by a circumferential force of F Newton. Which shown in fig.

The wheel is rotating at a speed of N R.P.M. WORKING Torque $T = F \times R N - m$ Angular speed of wheel $\omega = \frac{2\pi N}{60}$

So work done in one revolution = Force x Distance = $F \times 2\pi r$ joules

Mechanical Power developed $P_m = \frac{work \ done}{Time} = \frac{F \times 2\pi R}{\frac{60}{N}} = (F \times R) \times \frac{2\pi N}{60} = T \times \frac{\omega}{60}$ $P_m = T \times \omega = T \times \frac{2\pi N}{60}$

Mechanical power developed by the armature $P_M = E_b I_a$

$$\frac{2\pi NT}{60} = E_b I_a$$
$$T = \frac{E_b I_a}{2\pi N} \times 60$$

Put the value of $E_b = \frac{P\phi ZN}{60A}$





 $T = \frac{P\phi ZNI_a}{60A \times 2\pi N} \times 60 = \frac{P\phi ZI_a}{2\pi A}$







NUMERICAL PROBLEMS

Problem No.1 A six pole lap wound D.C. generator has 720 conductors a flux of 40 mwb per pole is driven at 400 rpm. Find the generator emf. (R.G.P.V. June-2017)

Solution- Given Data-				
No. of poles	P = 6			
For lap winding	A = P			
Speed of motor	N = 400 r.p.m.			
Flux	$\phi = 40 mwb = 40 \times 10^{-3} wb$			
Z = 720				
Find – Generated emf $E_g = \frac{NP\phi_z}{60A} = \frac{400 \times 6 \times 40 \times 10^{-3} \times 720}{60 \times 6} = 192V$				

Problem2- calculate the generated emf of a 8 pole wave wound D.C. generator which is having 720 conductors a flux per pole is 40 mwb and is driven at 400 rpm.

		(R.G.P.V. May-2018)
Solution- Given Data- 🎽 👘		
No of poles	P = 8	
No of conductor	Z = <mark>72</mark> 0	
Flux per pole	$\phi = 40 mwb = 40 \times 10^{-3} wb$	
No of parallel path for wave winding	A = 2	

Find -
$$E_g = \frac{NP\phi_z}{60A} = \frac{400 \times 8 \times 40 \times 10^{-3} \times 720}{60 \times 2} = 768V$$
 ans. COLLEGES
"WORKING TOWARDS BEING THE BEST"

Problem3- A shunt generator delivers 50 kw at 250 V and 400 r.p.m. The armature and field resistance are 0.02 ohm and 50 ohm respectively. Calculate the speed of the machine running as shunt motor and taking 50 KW input at 250 V.

V = 250 V

 $R_f = 50 \Omega$

 $N_1 = 400 \text{ R.P.M.}$ $R_a = 0.02 \Omega$

 $P = 50 kw = 50 \times 10^3 w$

Given Data-

Power of motor & generator Supply voltage & terminal voltage Speed of motor Armature resistance Field Resistance

Find – Speed of Machine $\frac{N_2}{N_1} = \frac{E_2}{E_1} \times \frac{\phi_1}{\phi_2}$

[shunt field current of motor & generator are constat so $\phi_2 = \phi_1$]

$$N_2 = \frac{E_2}{E_1} \times N_1 = \frac{246.1}{254.1} \times 400 = 387.41 \, R.P.M.$$





SM

$$\begin{bmatrix} N_{1} = 400 \ r.p.m. \\ Generator e.m.f. \ E_{1} = V + I_{a}R_{a} + brush \ drop \\ E_{1} = 250 + 205 \times 0.02 = 254.1 \\ V = 250, \ R_{a} = 0.02, brush \ drop = 0 \\ Armature \ current \ I_{a} = I_{L} + I_{sh} = 200 + 5 = 205 \\ I_{L} = \frac{P}{V} = \frac{50 \times 10^{3}}{250} = 200, \ I_{sh} = \frac{V}{R_{sh}} = \frac{250}{50} = 5 \end{bmatrix}$$

Generator e.m.f. $E_2 = V - I_a R_a - brush drop$ $E_2 = 250 - 195 \times 0.02 - 0 = 246.1$ $V = 250, R_a = 0.02, brush drop = 0$ Armature current $I_a = I_L - I_{sh} = 200 + 5 = 195$ $I_L = \frac{P}{V} = \frac{50 \times 10^3}{250} = 200, \ I_{sh} = \frac{V}{R_{sh}} = \frac{250}{50} = 5$

Problem.4- A 30 kw, 33 V D.C. shunt generator has armature and field resistance of 0.05 ohm and 100 ohm respectively. Calculate the total power developed by the armature when it delivers full output power. (R.G.P.V. MAY 2019)

Given Data-

Power Terminal voltage $R_a = 0.05 ohm$ Armature resistance $R_f = 100 ohm$ ARDS Field resistance

Find-

Total Power Developed by Generator $P = E_g I_a = 78.47 \times 909.42 = 71362.18 = 71.36 \ kw$

P = 30 kw V= 33 V

V= 33 V

$$\begin{bmatrix} I_a = I_L + I_{sh} = 909.09 + 0.33 = 909.42A \\ I_L = \frac{P}{V} = \frac{30 \times 10^3}{33} = 909.09, \ I_{sh} = \frac{V}{R_{sh}} = \frac{33}{100} = 0.33 \end{bmatrix}$$